Structure of Matter - I
March 31, 2016

PROBLEM 1. On electronic structure [16 pnts]

Consider an excited Cr^{3+} ion ($1s^2 2s^2 2p^6 3s^2 2p^6 3d^2 5d$).

a) Calculate the angle between the angular momentum vector and the z axis for a single 5d electron with m=-1. [1 pnt]

b) Sketch the radial part of this 5d wave function (rR_{5d}). [2 pnts]

c) Calculate the binding energy of hydrogenic Be^{3+}(5d). [1 pnt]

d) Explain why the excited Cr^{3+}(.....5d) is equally strong, stronger or weaker bound than a 5d electron in Be^{3+}. [2 pnt]

e) The excited Cr^{3+}(...$3d^2$5d) ion can decay via 2 pathways to its ground electronic configuration Cr^{3+}(...$3d^3$): P1 - $5d \xrightarrow{2x10^8} 4p \xrightarrow{2x10^7} 3d$ and P2 - $5d \xrightarrow{5x10^7} 4f \xrightarrow{7x10^7} 3d$. The transition rates (in s^{-1}) are indicated above the arrows. Calculate the probability that the ion decays via P1? [1 pnt]

f) Calculate the probability that after 10 ns the ion is still in the ...5d state. [2 pnts]

Now, consider Cr^{3+} in its ground electronic configuration $1s^2 2s^2 2p^6 3s^2 2p^6 3d^3$.

g) Determine the ground term of Cr^{3+}. [2 pnts]

h) Determine the ground level of Cr^{3+}. [1 pnt]

i) This Cr^{3+} ion has a nuclear spin of I=7/2. Due to the nuclear spin the ground level splits up into hyperfine levels. Determine all the possible hyperfine levels. [2 pnt]

Back to the excited Cr^{3+} ion.

j) Determine the ground term of the ...$3d^2$5d excited Cr^{3+} ion. Assume that Hund's rules apply. [2 pnts]

PROBLEM 2. On nuclear structure [14 ptn]

Consider the chromium isotope $^{53}_{24}Cr$.

a) Calculate the charge density [in units of elementary charge per fm^3] of the nucleus. [1 ptn]

b) Determine the nuclear spin J and the parity of the nucleus. [3 ptn]
Hint: Generic sequence of nuclear shell filling: 1s, 1p, 1d, 2s, 1f, 2p, 1g, ...

c) In reality the nuclear spin of this isotope is 3/2 what does this imply with respect to the energies of the nuclear shells. [2 ptn]

d) The Cr isotope in problem 1 was given to have I=7/2. What is the mass of that isotope. [2 pnts]

e) Assume that the $^{53}_{24}Cr$ isotope consists fully out of antimatter (antiprotons and antineutrons). Determine the nuclear spin J and the parity of this antimatter nucleus. [2 pnts]

Consider a nucleus X consisting of 29 protons and 47 neutrons.

f) Give the reaction formulae for α, β^+, and β^- decay. Label the daughter nucleus as Y. [2 pnts]

g) The binding energies of nuclei are well approximated by:

$$B = aA - bA^{2/3} - \frac{dZ^2}{A^{1/3}} - s\frac{(N-Z)^2}{A} - \frac{\delta}{A^{1/2}}$$

with a = 15.84 MeV, b = 18.33 MeV, d = 0.714 MeV, s = 23.2 MeV, and δ is 11, 0, and -11 MeV for odd-odd, even-odd, and even-even nuclei, respectively. Determine the difference in nuclear binding energy between the parent X and daughter Y after β^- decay. You may assume the constant d to be 0. [2 pnts]

PROBLEM 3. Elementary particles [10 pnts]

Consider a charmed Λ_c baryon with quark content udc which decays into $p + K^- + \pi^+$.

[quark compositions of π^+ : $u\bar{d}$ and K^-: $s\bar{u}$]

a) Determine the charge of this Λ_c baryon? [1 pnt]

b) Determine the hypercharge of this Λ_c baryon? [1 pnt]

c) Verify that the conservation laws for lepton and baryon number are respected? [1 pnt]

d) Which conservation law is violated? [1 pnt]

e) By which force(s) is the decay driven? [1 pnt]

f) What is the approximate time scale of the decay? [1 pnt]

g) Consider the J=3/2 family of charmed baryons. The 6 baryons of this family are: udc, usc, dsc, uuc, ddc, and ssc. Determine the hypercharge Y, the azimuthal isospin I_3, and the isospin of these six baryons. [2 pnts]

h) Why are baryons with two equivalent quarks (e.g. uuc) allowed? [1 pnts]

i) For the π^+, what is the color of the $\bar{d}$ quark if the color of the u quark is green? [1 pnts]